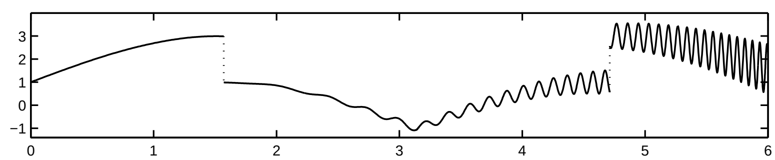
CHEBFUN AND CHEBOP

Lloyd N. Trefethen, Oxford University Computing Laboratory

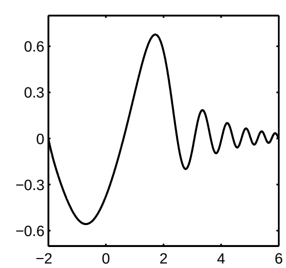
"Chebyshev technology", the use of Chebyshev series and interpolants for numerical computation, saw much of its early development in the 1950s and 1960s in England. Some of the contributions of those good old days include Lanczos's *Applied Analysis* with his Chebyshev tau method (Dublin, 1956), Clenshaw and Curtis's Chebyshev quadrature formula (NPL, 1960), Good's colleague matrices for Chebyshev zerofinding (NPL, 1961), Fox and Parker's survey of early steps toward Chebyshev spectral methods (Oxford, 1968) and Salzer's barycentric formula for interpolation in Chebyshev points (New York, 1972). Also important were Elliott, Mason, Wright....

It all sounds a bit old-fashioned. It shouldn't, though, for these methods are potentially even more important with today's computers than they were back then. Many readers of this newsletter may have attended a talk by me or my collaborators on the Chebfun system in object-oriented MATLAB, which puts these ideas into action in a way Clenshaw hardly dreamt of. If you execute `f=chebfun('2*abs(sin(x))+sign(cos(x))+.005*x.^3.*sin(x.^3)',[0 6])`, for example, the system works out in 1/4 second that this function can be represented to machine precision by a concatenation of four polynomial pieces of degrees 28, 48, 79 and 99:



Once you've got the chebfun you can find its integral with `sum(f)`, maximum with `max(f)`, roots with `roots(f)`, point values with (say) `f(0:6)` and so on, all at great speed to roughly machine precision. And you can carry out further compuations like `f.^2` or `exp(f)` with "symbolic feel and numerical speed".

The chebop system takes the next step and overloads linear operators within the same framework via lazy evaluation of spectral differentiation matrices (far from obvious!—no space to explain here). Thus for example if we execute the commands `[d,x]=domain(-2,6)`, `D=diff(d)`, `X=diag(x.^3)`, `L=D^2+D+X` , `L.bc='dirichlet'`, `u=L\1`, in about 0.1 secs. we get the solution to $u'' + u' + x^3u = 1$ with zero boundary conditions on $[-2, 6]$ (right).



We hope that chebfuns and chebops will prove a step towards computing as easily with functions in the 21st century as we learned to do with numbers in the 20th. To find out more, have a look at `www.comlab.ox.ac.uk/chebfun/`, and please get in touch if you have an application you'd like to explore together. My chebfun teammates are Toby Driscoll, Ricardo Pachón and Rodrigo Platte. Thanks also to Zachary Battles, Folkmar Bornemann and the EPSRC.